Welcome to Durham for Algebraic, Topological and Complexity Aspects of Graph Covers 2017!

This booklet contains the abstracts of the three invited lectures to be given by Pavol Hell, Daniel Král' and Edita Máčajová. You can also find the abstracts of the contributed talks and a schedule.

All of the lectures will take place in room E240 in the School of Engineering and Computing Sciences. The room (and others if requested) will be available throughout the workshop for delegates to meet and work. A buffet lunch will be provided in the adjacent atrium.

Wireless network access can be obtained using Eduroam.

Financial support for the workshop from the London Mathematical Society and Durham University is gratefully acknowledged.

Monday 9 January

0900		Registration
0930-0935		Welcome to Durham
0935-1030	**Pavol Hell**	**Obstruction Certificates for Geometrically Defined Graph (and Digraph) Classes**
1030-1100		Tea/Coffee
1100-1230		Open Problems
1230-1330		Lunch
1330-1500		Discussions
1500-1530		Tea/Coffee
1600-1700		Tour of Durham Castle
1700-1830		Welcome Reception, Palace Green Library, Deane Room

Tuesday 10 January

0930-1030	**Edita Máčajová**	**Point-line Configurations and Conjectures in Graph Theory**
1030-1100		Tea/Coffee
1100-1145	Marston Conder	Symmetric Cubic Graphs as Cayley Graphs
1145-1230	Barnaby Martin	The complexity of surjective homomorphism problems
1230-1330		Lunch
1330-1500		Discussions
1500-1530		Tea/Coffee
1530-1700		Discussions

Wednesday 11 January

0930-1015	Martin Skoviera	Cubic graphs that cannot be covered with four perfect matchings
1015-1100	Pavel Klavik	Graph Isomorphism Restricted by Lists
1100-1130		Tea/Coffee
		Excursion (packed lunch provided)

Thursday 12 January

0930-1030	**Daniel Král'**	**Subgraph Counts in Large Graphs**
1030-1100		Tea/Coffee
1100-1145	Thomas Bellitto	Complexity of locally-injective homomorphisms to tournaments
1145-1230	Peter Zeman	Automorphism groups of planar graphs
1230-1330		Lunch
1330-1500		Discussions
1500-1530		Tea/Coffee
1530-1700		Discussions

Friday 13 January

0930-1030		Discussions
1030-1100		Tea/Coffee
1100-1230		Solved Problems
		For delegates remaining in Durham, rooms will be available in the School for the rest of the day and lunch will be provided

Invited Lectures

Obstruction Certificates for Geometrically Defined Graph (and Digraph) Classes

Pavol Hell

In this talk I will focus on obstruction characterizations for interval graphs and their analogues, principally versions of interval bigraphs, interval digraphs, and circular arc-graphs. I will propose a new geometrically defined class of digraphs that generalizes the first three classes, and present an obstruction characterization for it. I will also present a separate obstruction characterization for the class of circular-arc graphs, that gives an answer to a question of Klee, and of Hadwiger and Debrunner, from the early 1960s. The first three classes are closely related to the complexity of the list homomorphism problem.

This is joint with T. Feder, J. Huang, and A. Rafiey for the interval graphs and their generalizations, and with M. Francis and J. Stacho for the circular arc graphs.

Subgraph Counts in Large Graphs

Daniel Král'

Many problems in extremal graph theory relate to understanding possible combinations of densities of subgraphs in large graphs. A recent breakthrough is Razborov's solution of the edge vs. triangle density problem (proving an old conjecture of Lovász and Simonovits, which has been extended by Nikiforov to complete graphs of order four and by Reiher to complete graphs of an arbitrary order. In this talk, we survey results in this area and report on our results on densities of 3-vertex graphs.

The talk contains results based on a joint work with Roman Glebov, Andrzej Grzesik, Ping Hu, Tamás Hubai and Jan Volec.

Point-line Configurations and Conjectures in Graph Theory

Edita Máčajová

Many open problems in Graph Theory can be reduced to the family of cubic graphs. Moreover, in a lot of cases it is enough to focus on an even smaller set of graphs — snarks, which are cubic graphs which do not admit a proper 3-edge-colouring.

A *configuration* $\mathcal{C} = (P, B)$ consists of a finite set P of *points* and a finite set B of blocks. *Blocks* are 3-element subsets of P such that for each pair of points of P there is at most one block in B which contains both of them.

A colouring of a cubic graph G with a configuration $\mathcal{C} = (P, B)$ is an assignment of an element from P to each edge of G such that the three points that meet at any vertex form a block of B.

During this talk we will discuss several well known conjectures and open problems from the view of colouring with configurations and thereby provide an unifying view of them. These problems include the Fulkerson Conjecture and the Petersen Coloring Conjecture, the Fan-Raspaud Conjecture, problems concerting the perfect matching index of a graph and others.

Contributed Talks

Complexity of locally-injective homomorphisms to tournaments

Thomas Bellitto

This talk studies locally-injective homomorphisms, also known as partial graph covers, between oriented graphs. Depending on the applications, several definitions of local injectivity exist in the literature that lead to subtly different problems. We study for each of them the complexity of the problem of determining the existence of a locally-injective homomorphism from a graph to a given target tournament. The specific case where the target graph is a tournament is an important step toward more general results and already answers several related questions such as the complexity of oriented locally-injective colouring. We find dichotomy theorems for the complexity of locally-injective homomorphisms to reflexive tournaments, and report on progress towards such a theorem for locally-injective homomorphisms to irreflexive tournaments.

Symmetric Cubic Graphs as Cayley Graphs

Marston Conder

A graph Γ is *symmetric* if its automorphism group acts transitively on the arcs of Γ, and *s-arc-transitive* if its automorphism group acts transitively on the set of s-arcs of Γ. Furthermore, if the latter action is sharply-transitive on s-arcs, then Γ is *s-arc-regular*.

It was shown by Tutte (1947, 1959) that every finite symmetric cubic graph is s-arc-regular for some $s \leq 5$. Djokovic and Miller (1980) took this further by showing that there are seven types of arc-transitive group action on finite cubic graphs, characterised by the stabilisers of a vertex and an edge. The latter classification was refined by Conder and Nedela (2009), in terms of what types of arc-transitive subgroup can occur in the automorphism group of Γ.

In this talk we consider the question of when a finite symmetric cubic graph can be a Cayley graph. We show that in five of the 17 Conder-Nedela classes, there is no Cayley graph, while in two others, every graph is a Cayley graph. In eight of the remaining ten classes, we give necessary conditions on the order of the graph for it to be Cayley; there is no such condition in the other two. Also we use covers (and the 'Macbeath trick') to show that in each of those last ten classes, there are infinitely many Cayley graphs, and infinitely many non-Cayley graphs.

This research grew out of some recent discussions with Klavdija Kutnar and Dragan Marusic.

Graph Isomorphism Restricted by Lists

Pavel Klavik

The complexity of graph isomorphism (GraphIso) is a famous unresolved problem in theoretical computer science. For graphs G and H, it asks whether they are the same up to a relabeling of vertices. In 1981, Lubiw proved that list restricted graph isomorphism (ListIso) is NP-complete: for each $u \in V(G)$, we are given a list $\mathfrak{L}(u) \subseteq V(H)$ of possible images of u. After 35 years, we revive the study of this problem and consider which results for GraphIso translate to ListIso.

We prove the following:

1. When GraphIso is GI-complete for a class of graphs, it translates into NP-completeness of ListIso.
2. Combinatorial algorithms for GraphIso translate into algorithms for ListIso: for trees, planar graphs, interval graphs, circle graphs, permutation graphs, bounded genus graphs, and bounded treewidth graphs.
3. Algorithms based on group theory do not translate: ListIso remains NP-complete for cubic colored graphs with sizes of color classes bounded by 8.

Also, ListIso allows to classify results for the graph isomorphism problem. Some algorithms are robust and translate to ListIso. A fundamental problem is to construct a combinatorial polynomial-time algorithm for cubic graph isomorphism, avoiding group theory. By the 3rd result, ListIso is NP-hard for them, so no robust algorithm for cubic graph isomorphism exists, unless P = NP.

The complexity of surjective homomorphism problems

Barnaby Martin

Classifying the complexity of surjective homomorphism problems seems to be a more challenging task than that for normal homomorphism problems. While H-Colouring is well-understood for graphs H, Surjective H-Colouring is far from being classified. We survey known results about surjective homomorphism problems, with reference to the related Compaction and Retraction problems, look at recent work in the area, and ponder future directions for study.

Cubic graphs that cannot be covered with four perfect matchings

Martin Skoviera

The celebrated Berge-Fulkerson conjecture suggests that every bridgeless cubic graph can have its edges covered with at most five perfect matchings. Since three perfect matchings suffice if and only if the graph in question is 3-edge-colourable, uncolourable cubic graphs fall into two classes: those that can be covered with four perfect matchings, and those that require at least five. Cubic graphs that cannot be covered with four perfect matchings are extremely rare. Among the 64326024 snarks (uncolourable cyclically 4-edge-connected cubic graphs with girth at least five) on up to 36 vertices there are only *two* graphs that cannot be covered with four perfect matchings - the Petersen graph and a snark of order 34.

In this talk we show that coverings with four perfect matchings can be described in terms of flows whose values lie in the configuration of six lines spanned by four points of the 3-dimensional projective geometry $PG(3,2)$ in general position. This characterisation provides us with a convenient tool for constructing new families of snarks that cannot be covered with four perfect matchings. One of our families includes all snarks currently known to require five perfect matchings to cover their edges. Another construction is based on regular covering projections using voltage graphs with voltages in $\mathbb{Z}_5$.

This is a joint work with Edita Máčajová.

Automorphism groups of planar graphs

Peter Zeman

In 1975, Babai characterized which abstract groups can be realized as the automorphism groups of planar graphs. In this paper, we give a more detailed and understandable description of these groups. We describe stabilizers of vertices in connected planar graphs as the class of groups closed under the direct product and semidirect products with symmetric, dihedral and cyclic groups. The automorphism group of a connected planar graph is then obtained as a semidirect product of a direct product of these stabilizers with a spherical group. Our approach translates into a quadratic-time algorithm for computing the automorphism group of a planar graph which is the first such algorithm described in details.

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