Point-line configurations and conjectures in Graph Theory

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ATCAGC 2017, Durham, January 2017

Introduction

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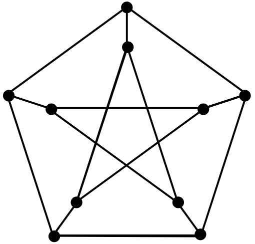
- cubic graphs
  - 3-edge-colourabe
  - snarks – cubic graphs that do not admit a 3-edge-colouring



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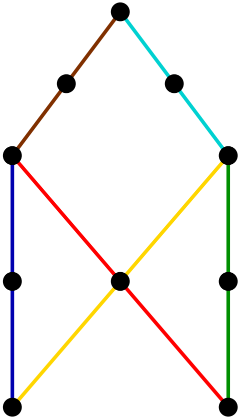
- snarks are crucial for many conjectures and open problems
- sometimes useful: use more than 3 colours and specify the allowed triples
- configuration $\mathcal{C} = (P, B)$
  - P – finite set of points
  - B – finite set of blocks (3-element subsets of P such that for each pair of points of P there is at most one block of B which contains both of them)

Example: a configuration



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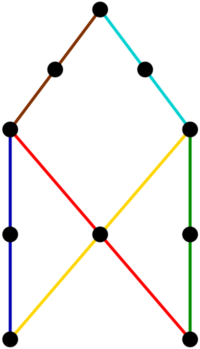
Question

For which cubic graph G and configuration $\mathcal{C}$ there exists a $\mathcal{C}$-colouring of G?

- a configuration $\mathcal{C}$ is universal if every cubic graph has $\mathcal{C}$-colouring
- a configuration $\mathcal{C}$ is bridgeless universal if every *bridgeless* cubic graph has $\mathcal{C}$-colouring

Example: a configuration



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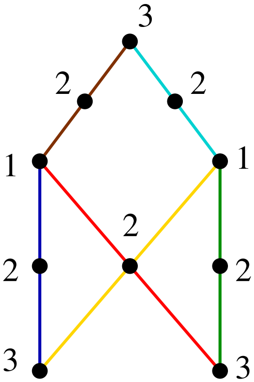

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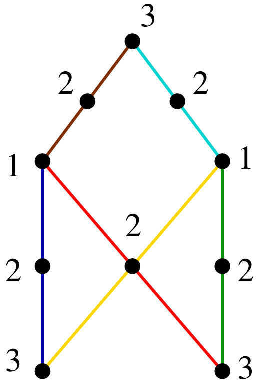
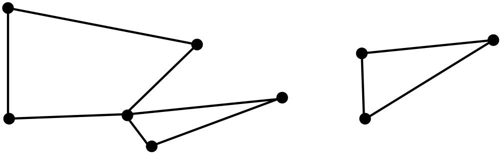


this configuration is not universal nor bridgeless-universal

Definitions

cycle – a graph in which every degree is even



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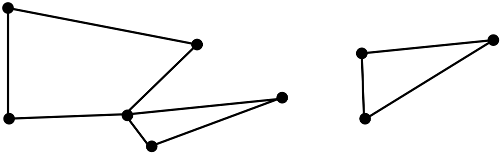
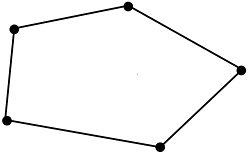


simple cycle – 2-regular connected graph



5-Cycle Double Dover Conjecture (5CDCC)

Conjecture (Celmins 1984, Preissmann 1981)

Every bridgeless graph admits a collection of five cycles such that each edge belongs to exactly two of them.

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- equivalent to its restriction on cubic graphs
- true for all 3-edge-colourable graphs

Reduction to cubic graphs

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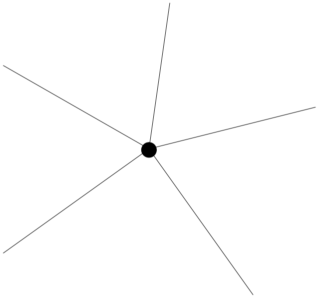

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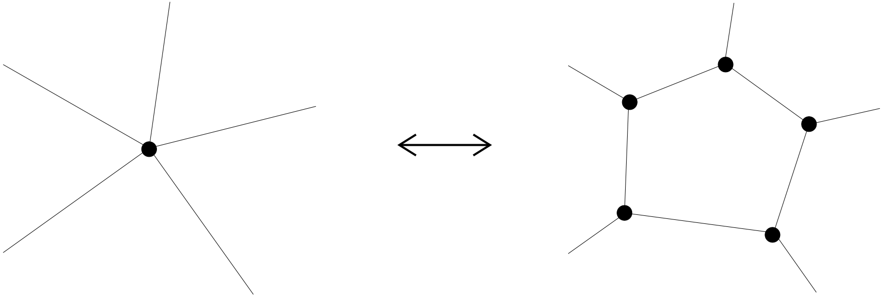

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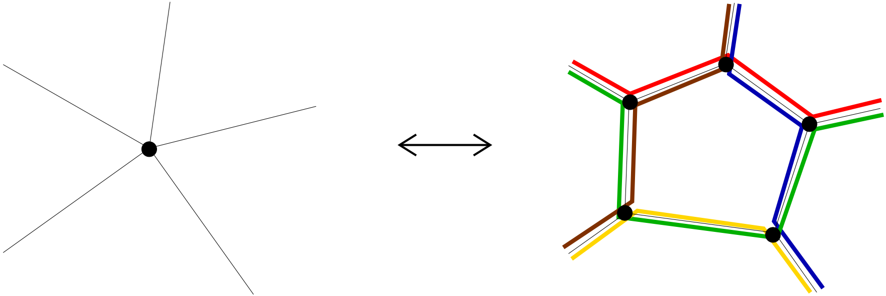

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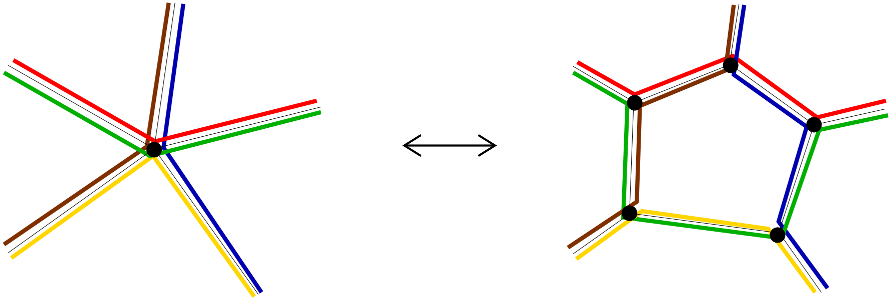
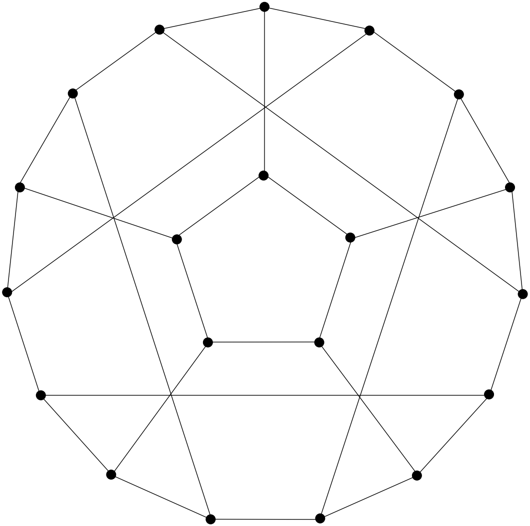


Cycle double cover of I_5



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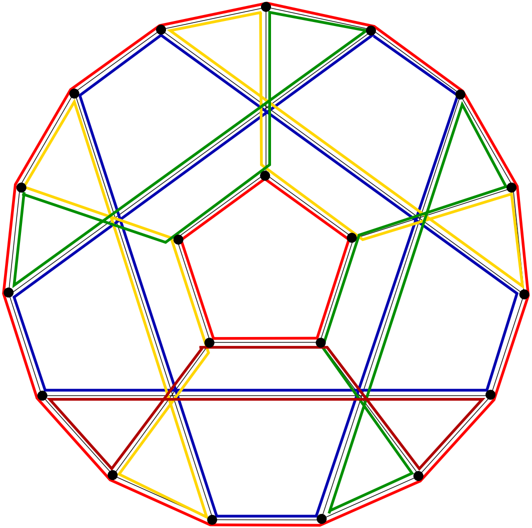

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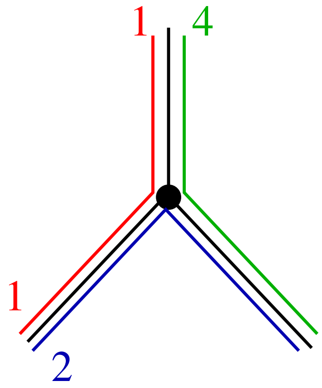

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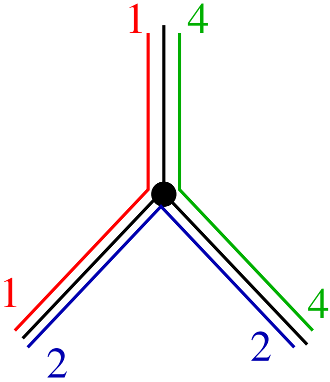

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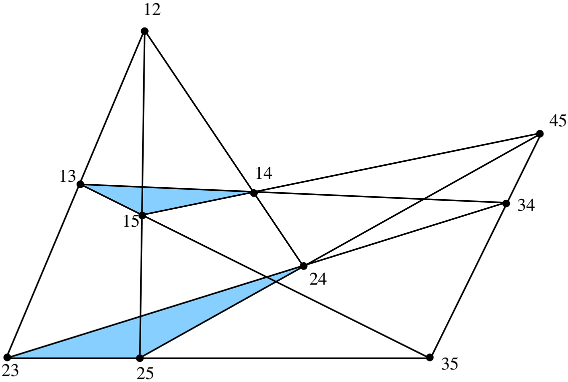

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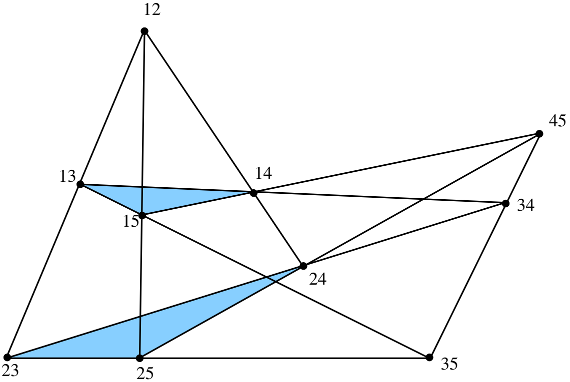


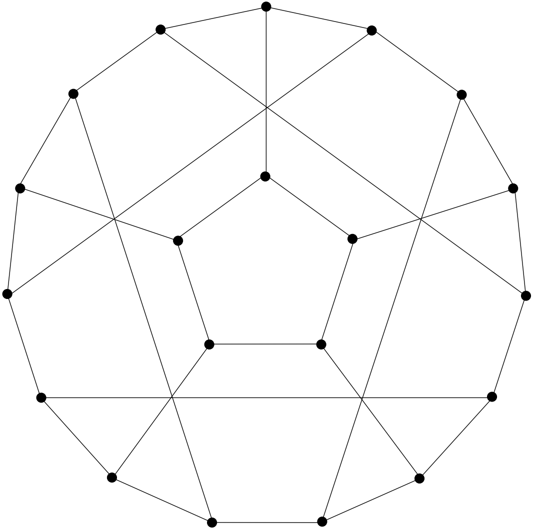
Desargues Configuration

Fulkerson Conjecture

Fulkerson Conjecture (Berge, Fulkerson, 1971)

Every bridgeless cubic graphs contains a family of six perfect matchings that together cover each edge exactly twice.

6 perfect matchings on I_5



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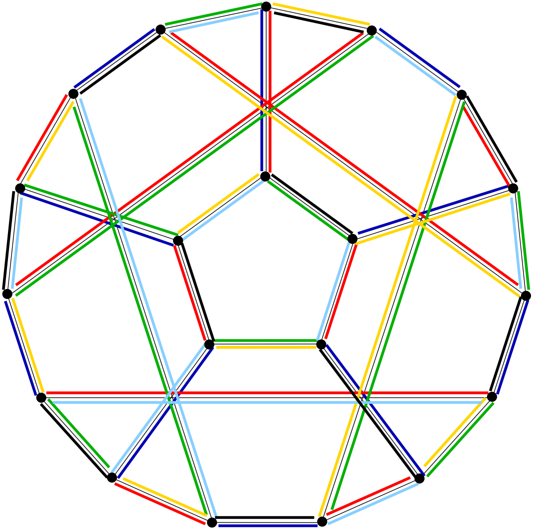
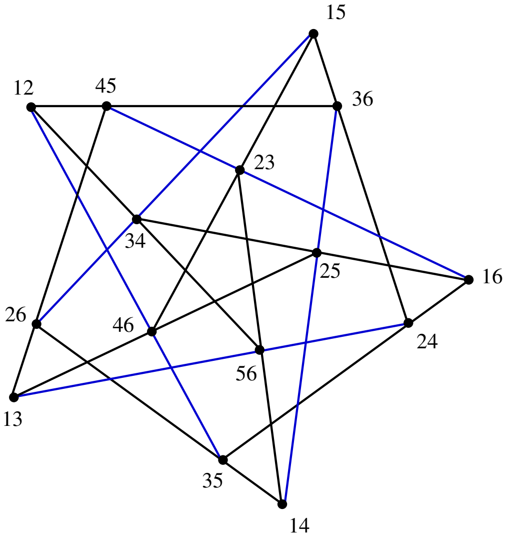


Cremona–Richmond configuration



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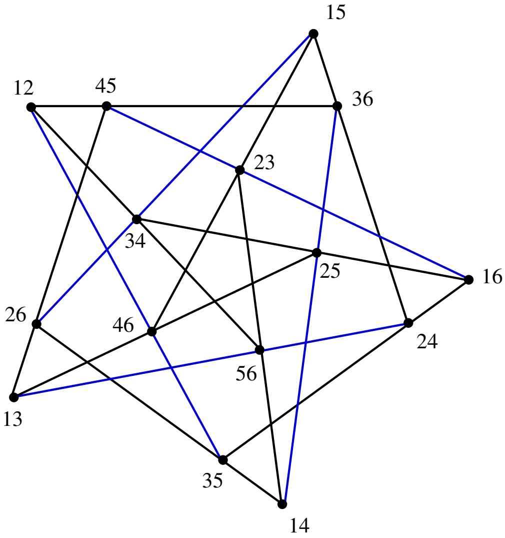
6 perfect matchings on I_5

Petersen Colouring Conjecture (Jaeger, 1988)

The edges of every bridgeless cubic graph G can be mapped into the edges of the Petersen graph in such a way that any three mutually incident edges of G are mapped to three mutually incident edges of the Petersen graph.

Cremona–Richmond configuration



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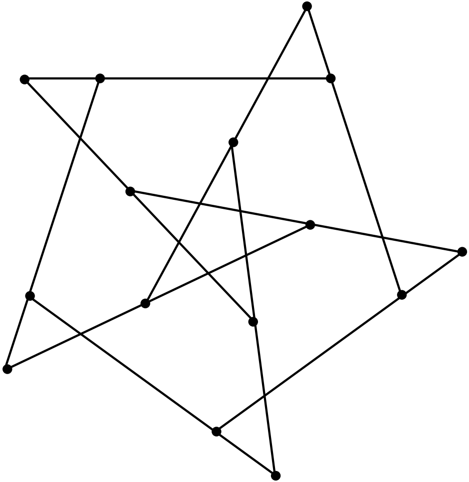

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- Fulkerson Conjecture $\Rightarrow$ Berge Conjecture

Berge Conjecture $\Leftrightarrow$ Fulkerson Conjecture

Theorem (Mazzuoccolo, 2011)

The Berge Conjecture and the Fulkerson Conjecture are equivalent.

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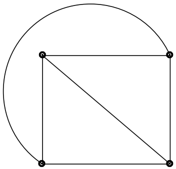

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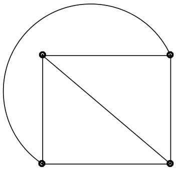
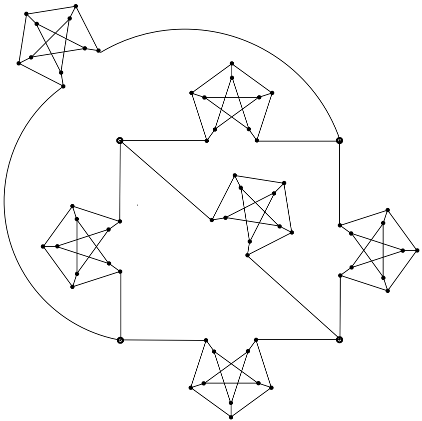
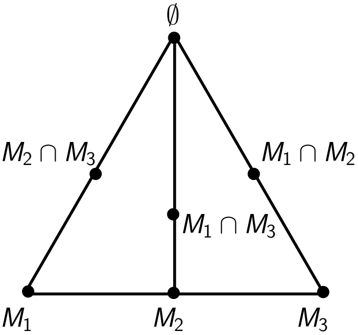




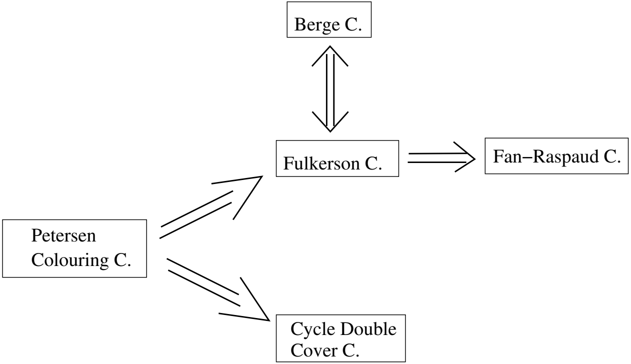
Fan-Raspaud Conjecture

Fan-Raspaud Conjecture, 1994

Every bridgeless cubic graph has three perfect matchings with empty intersection.

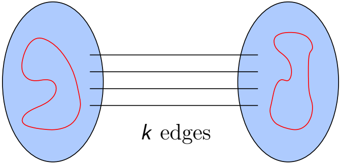


Implications



Definitions

Cyclic connectivity is the smallest number of edges which have to be removed in order to obtain at least two components containing cycles



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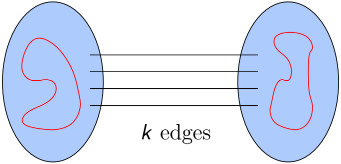

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There are no snarks with cyclic connectivity greater than 6.

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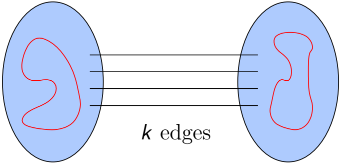

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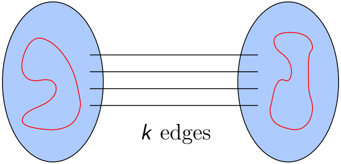

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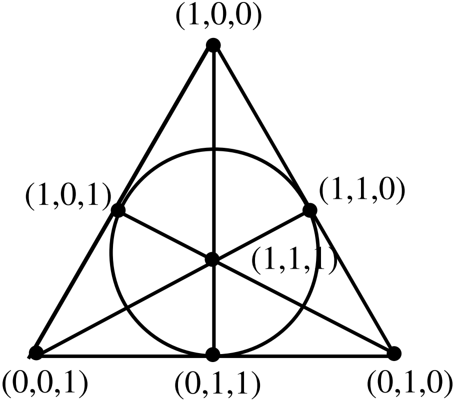
- $\xi(G) = 0 \Leftrightarrow G$ is 3-edge-colourable

Minimal counterexample

conj.	girth	cyclic connectivity	oddness
Petersen C. Conjecture	≥ 4	≥ 3	≥ 2
5 Cycle Double Cover C.	≥ 12 [Huck]	≥ 4	≥ 6 [Huck]
Fulkerson Conjecture	≥ 4	≥ 3	≥ 2
Fan–Raspaud Conjecture	≥ 4	≥ 3	≥ 4 [EM, Skov]

Fano Plane



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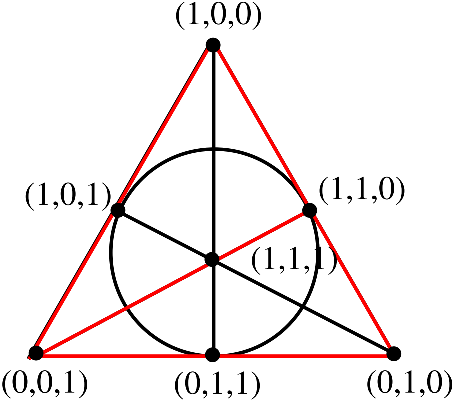

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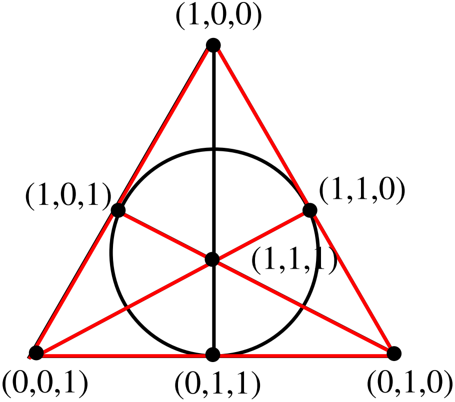

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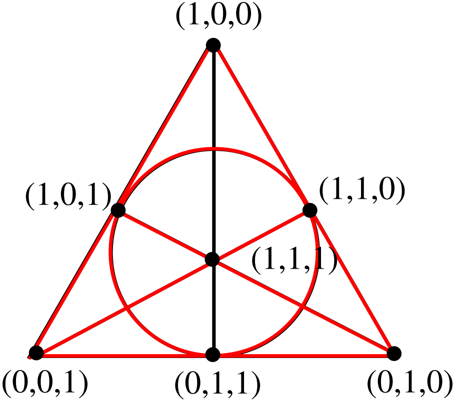
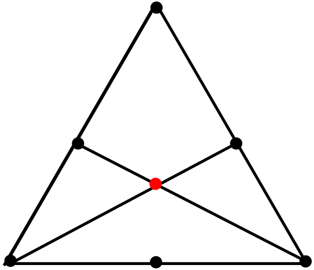


F_6-configuration is bridgeless universal [EM,Škoviera]

$F_5 \Leftrightarrow A_5$



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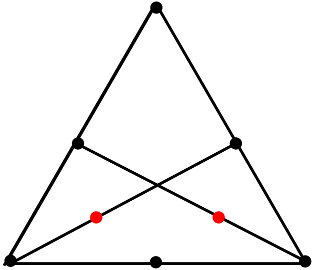

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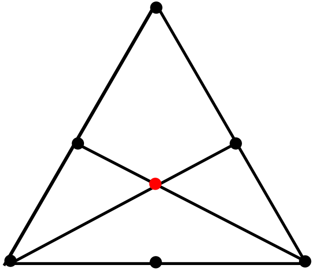
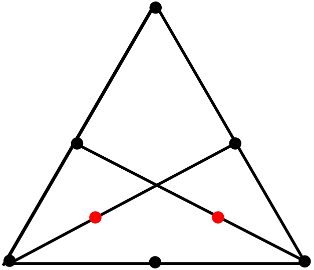
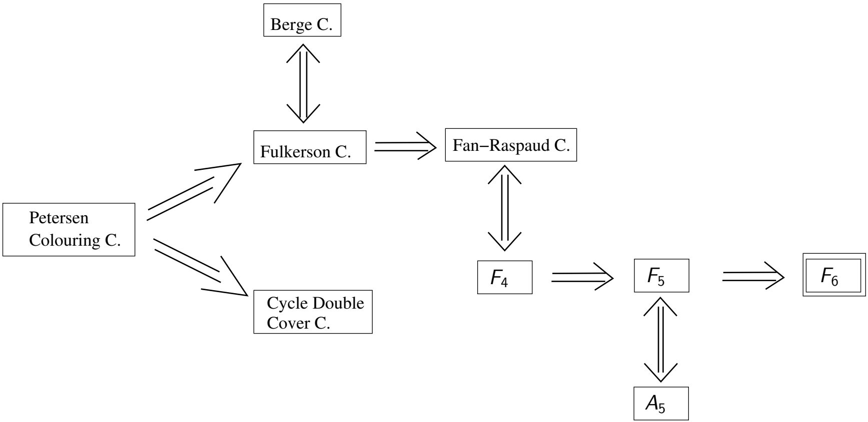




Theorem (Král', EM, Pangrác, Raspaud, Sereni, Škoviera)

A cubic graph is F_5-colourable $\Leftrightarrow$ it is A_5-colourable.

Implications



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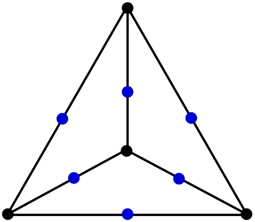
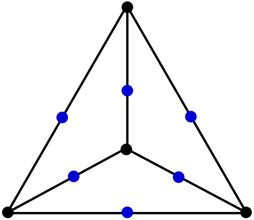


- smallest example $G = K_4$

"K_4"-configuration and four perfect matchings



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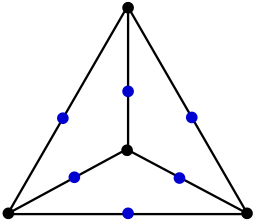

configuration $\mathcal{T}$

- 10 points, 6 blocks
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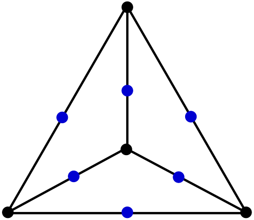

configuration $\mathcal{T}$

- 10 points, 6 blocks
- this configuration is not 3-colourable
- this configuration appears in a different context:

Theorem (EM,Škoviera, 2017+)

A cubic graph G is $\mathcal{T}$-colourable $\Leftrightarrow$ the edges of G can be covered by at most 4 perfect matchings.

Perfect matching covers of cubic graphs

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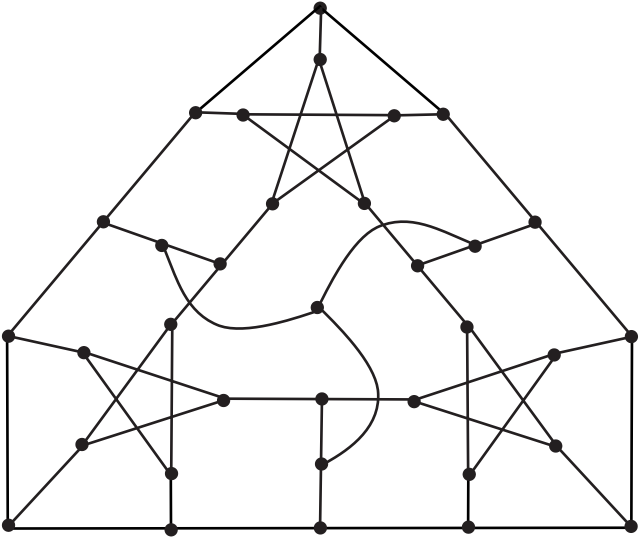
A snark of order 34 with $\tau(G) = 5$



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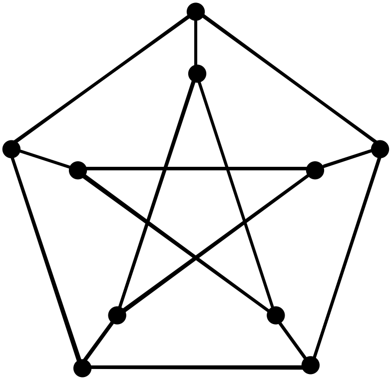

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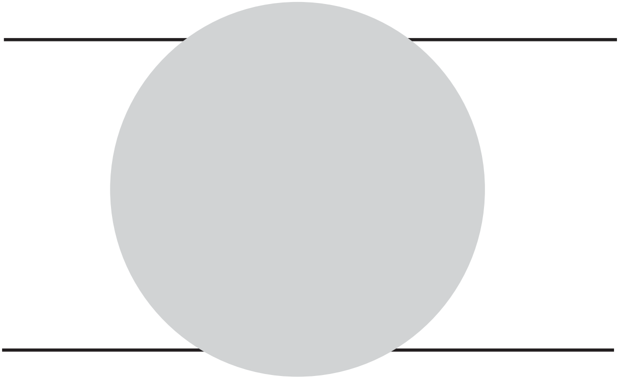

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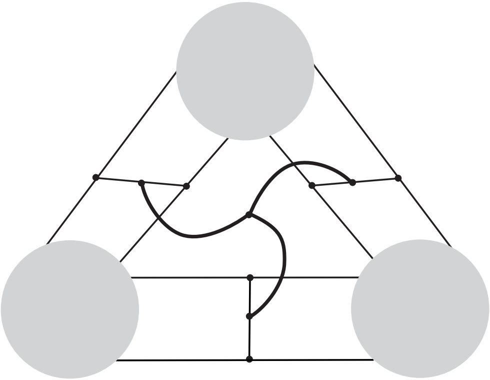

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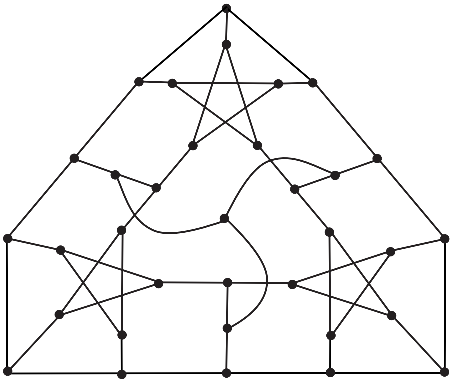

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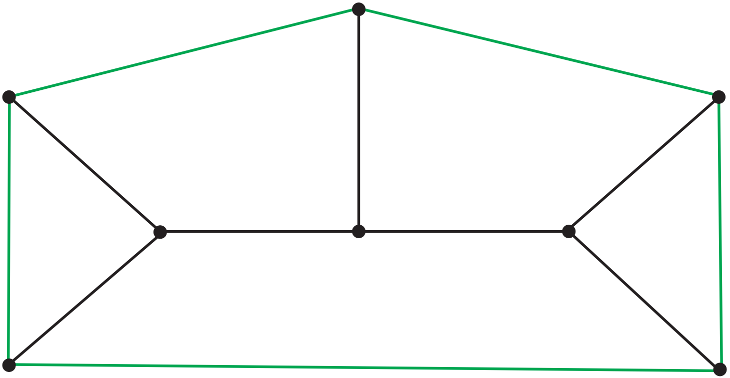

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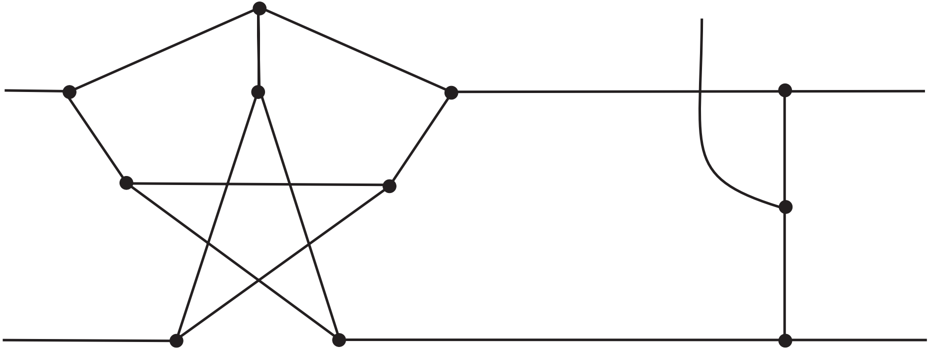

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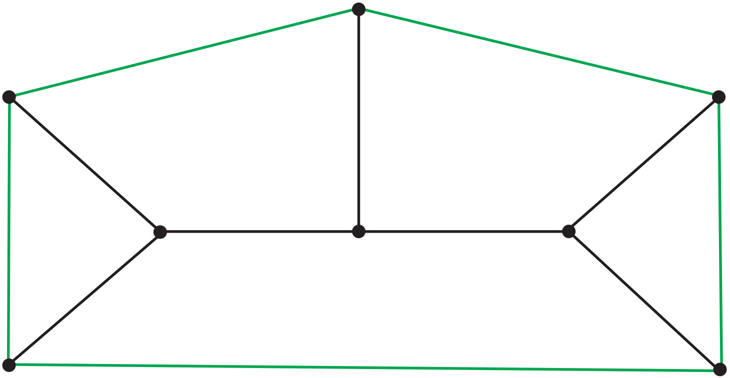

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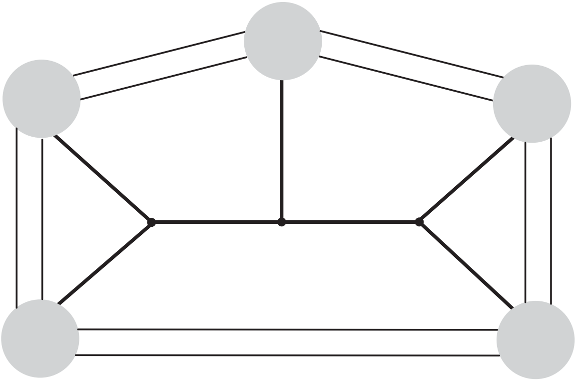

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- Treelike snarks have a more general shape than windmill snarks.

However:

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- Proofs heavily depend on computer-aided arguments.

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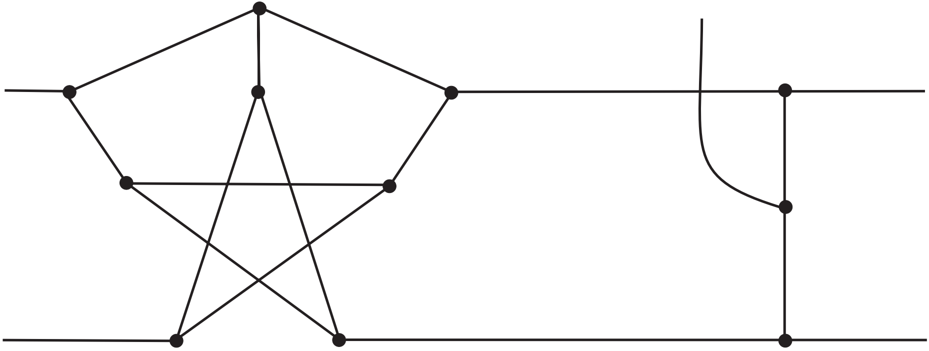

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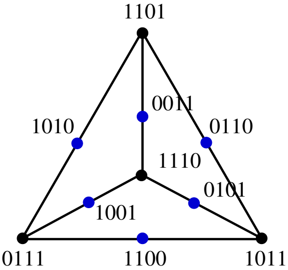
8.1 The pattern set of the Petersen fragment

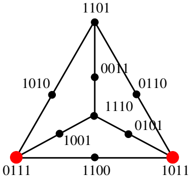
The pattern set of F_0 (42 patterns):

A A AB AC AD	A B CD AB AB	A BC D BC BC
A A AB C D	A B CD AC AC	A BC D BD BD
A AB A AC AD	A B CD C C	A BC D D D
A AB A BC BD	A B CD CD CD	AB AB AB AC AD
A AB AC A AD	A BC A AB BD	AB AC AB AB AD
A AB AC B BD	A BC B AB AD	AB AC AB BC CD
A AB AC C CD	A BC B BC CD	AB AC AD A A
A B AB AB CD	A BC BD A AB	AB AC AD AB AB
A B AB AC BD	A BC BD BC C	AB AC AD AD AD
A B AC A D	A BC BD BD D	AB AC AD B B
A B AC AB BD	A BC D A A	AB AC AD BC BC
A B C A AD	A BC D AB AB	AB AC AD BD BD
A B C C CD	A BC D AD AD	AB AC AD D D
A B CD A A	A BC D B B	AB CD AC AB BC

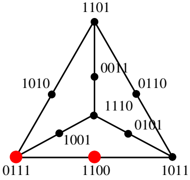
Tetrahedral $\mathbb{Z}_2^4$-flow



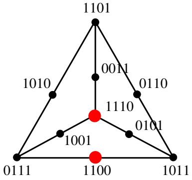
Types of connectors



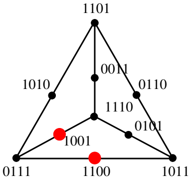
edge



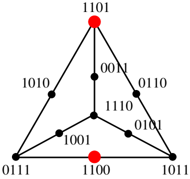
half–line



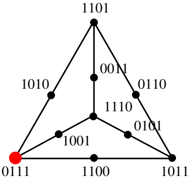
altitude



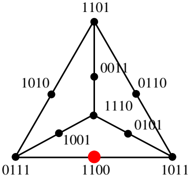
angle



axis



zero



zero

Set of transitions $\mathcal{M}$

Let $\mathcal{M}$ be the set of all transition through a $(2, 2; 1)$-pole containing all the transition of the following types:

- axis $\xrightarrow{1}$ half-line
- edge $\xrightarrow{1}$ half-line
- zero $\xrightarrow{1}$ half-line
- angle $\xrightarrow{1}$ half-line
- angle $\xrightarrow{1}$ altitude
- altitude $\xrightarrow{1}$ edge
- axis $\xrightarrow{2}$ edge
- edge $\xrightarrow{2}$ edge
- edge $\xrightarrow{2}$ zero
- zero $\xrightarrow{2}$ edge
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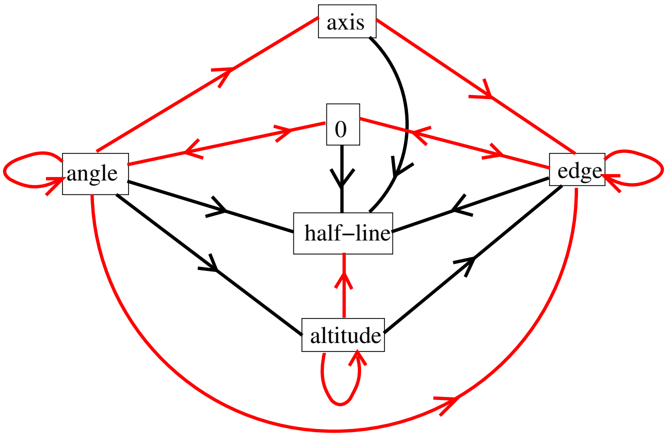

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Theorem (EM, Škoviera, 2017+)

- *Let S be a $(2,2)$-pole created from a snark G with $\tau(G) \geq 5$ by removing two adjacent vertices. Then $\mathbf{T}(S \circ I) \subseteq \mathcal{M}$.*

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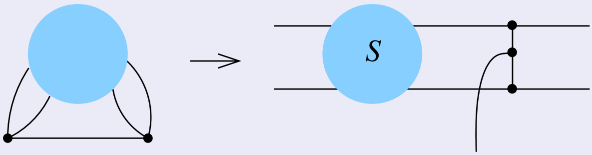

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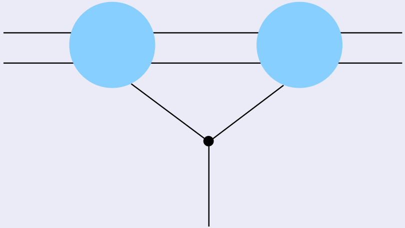

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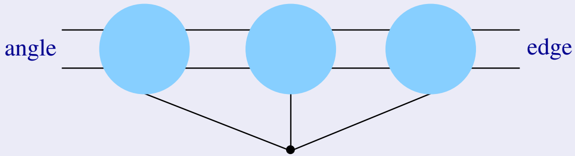

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Theorem (Král', EM, Pór, Sereni, 2010)

Every non-projective, non-affine point-transitive STS is universal.

Thank you!